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Causal Probabilistic Models for Evaluating Future Transoceanic Airlane Separations

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With the advent of automatic dependent surveillance (ADS), a detailed probability model of aircraft cross-track deviations is required to determine the impact of ADS. A suitable causal probability model is presented where normal navigation, degradation, pilot blunder, and failure are each modeled by Gaussian density functions with mean and standard deviations defined by the physics of the event. The overall model which is a weighted sum of these Gaussian error probabilities is thus amenable to extrapolation. Overlap and encroachment probabilities are derived, and the impact of ADS on this model determined. It is shown that by using a simple form of ADS, separation standards can be reduced and by transmitting a figure of merit (FOM) providing information on failures and degradations, these separation standards can be further reduced. The results suggest an improvement by a factor of two from current separation standards.

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I. INTRODUCTION

In areas of the world where conventional radar coverage may not be possible, for example, over the ocean, organized track separations standards have usually been established and justified using the Reich collision risk model [3–5]. The probability density functions (pdf's) associated with aircraft track errors are the critical parts in the model. After extensive empirical studies, the data have been fit very closely to double, double exponential density function given by

$$f(x) = (1-\alpha)\frac{\exp\left[-\frac{\sqrt{2}|x|}{\sigma_A}\right]}{\sqrt{2}\sigma_A} + \alpha\frac{\exp\left[-\frac{\sqrt{2}|x|}{\sigma_B}\right]}{\sqrt{2}\sigma_B} \tag{1}$$

with $\sigma_A = 4$ nm, $\sigma_B = 73$ nm, and $\alpha = 0.00106$ have been used for the North Atlantic.

The non-Gaussian tails which drive the collision risk is the aggregate effect of pilot blunders, equipment degradations and failures, and poor quality navigation systems improperly entering the airspace. Because there are only three parameters σ_A, σ_B, and α in this model, it is difficult to extrapolate the parameters to reflect changes in mix of navigation systems, changes in procedures, and other details.

Two major events which will take place in the next five years or so will radically change the oceanic flying environment. One is the availability of automatic dependent surveillance (ADS). This is a satellite-based communications service which will make the sending of rapid automatic position reports to the Air Traffic Control Centers (ATC) centers possible, and subsequent control when necessary from these centers. The second is the inclusion of a significant fraction of Global Positioning Satellite (GPS) receivers in the mix of navigation systems.

The use of ADS can eliminate some of the errors but not others. The result is lower magnitude tails, which can lead to lower separation standards at the same level of safety. When a significant portion of the navigation mix uses GPS, it is possible that further reduction on separations standards can be achieved. Again, the double, double exponential pdf (see eqn. (1)) with only three parameters does not provide sufficient flexibility to evaluate accurately the impact of these new developments.

An improved probability model is presented here which is amenable to extrapolation. It is based on the hypothesis that each type of normal navigation, and each type of blunder (equipment-induced or pilot-induced) is characterized by one or more Gaussian density functions with associated means and standard deviations defined by the event. The overall density function is made up of a weighted sum of these Gaussian density functions. The weights associated with the density functions are determined from available data on mix of navigation systems, reliability data, and data on blunders. The model is causal in that each term is related to a specific event in a given navigation system. Thus, if a specific event is eliminated or modified because of application of ADS, or if a new mix of navigation systems is present in the airspace, then the new pdf can be calculated from the proposed model. New separation standards can then be computed.

Section II of this paper develops the general functional form of the causal probability model taking into account normal and degraded navigation errors, specific types of pilot blunders, reliability and mix of navigation system types. Section III discusses how the models for these individual classes of errors follow from the general form.

Section IV develops the modeling procedures involved in factoring in the effects of ADS. Such factors as sampling rate and transmission of a figure of merit (FOM) indicating degradation and failure are included.

The resultant model is quite detailed, involving the specification of many parameters. Section V describes by examples a rational approach toward defining these parameters. The North Atlantic Minimum Navigation Performance Specification (MNPS) airspace is studied. Blunder data for the years 1983 through 1985, FAA data on the mix of navigation systems and number of flights, and manufacturer data are used in determining these parameters.

Section VI presents numerical results for the MNPS. First, the credibility of the model is established by showing that the probability density derived from the model and the above-mentioned data are a very close fit of the double, double exponential pdf derived from the empirical data, (eqn. (1)). Then the model is used to determine suggested lateral separation standards for various levels of complexity of ADS. It is found that suggested lateral separations can be reduced from the current 60 nm to around 40 nm if the simplest form of ADS (reporting position) is used. If, in addition, an FOM containing information on failures and degradations (based on a simple degradation detection technique) is included, suggested separations standards can be further reduced to about 30 nm. Sampling position faster than every 0.1 h does not generally provide any improvement for the current navigation system mix.

Section VII presents summary and conclusions.

II. GENERAL FUNCTIONAL FORM OF CAUSAL PROBABILITY MODEL

The main classes of characteristics considered in formulating the general model are: normal and degraded navigation systems, reliabilities, pilot blunders and mixes of different types of navigation systems. The main assumptions in forming the model are the following.

1. All navigational degradations and navigation system failure are mutually exclusive so that the individual probabilities add.
2. The joint probability of two different blunders occurring is negligible. Hence, blunders also are considered mutually exclusive and the blunder probabilities add.
3. Navigation errors as well as navigation failure and blunders are considered independent. Hence, the probability of the joint event is the product of the probabilities of individual events. However, the joint probability of a navigation degradation and a blunder is considered negligible and hence the product of these probabilities are also negligible.

A. Navigation Errors

From assumption 1, the total pdf associated with a navigation system L is a sum of individual Gaussian density functions as shown in (2), with the parameters being implicit functions of time.

$$f_{pL}(x) = P_{NL}\frac{1}{\sqrt{2\pi}\sigma_{NL}}\exp\left[-\frac{x^2}{2\sigma_{NL}^2}\right] + \sum_{j=1}^{K}\alpha_{Lj}\frac{1}{\sqrt{2\pi}\sigma_{pLj}}\exp\left[-\frac{(x-m_{pLj})^2}{2\sigma_{pLj}^2}\right]. \quad (2)$$

In the above equation the first term represents the density function associated with normal navigation and the other terms represent various conditions of degradations such as lane slip in an OMEGA system or gyro degradation in an inertial navigation system. The variable P_{NL} is the probability of normal navigation which is close to 1, and σ_{NL}, which may be an implicit function of time, is the standard deviation associated with normal navigation. Subscript N represents normal operation and K represents the total number of degradations. The means of all these density functions are assumed to be zero. The probability that the jth degradation will occur (at a specified time) is α_{Lj} and the associated standard deviation is σ_{pLj}. For most cases the mean m_{pLj} will be zero. The index p represents position and the running index j either represents the various types of degradations (e.g., lane slip and sudden ionospheric disturbance for an OMEGA system) or the degradation of an instrument at different specific times prior to the current time T (e.g., gyro degradation 1/2 h and 1 h before the current time in an inertial navigation system).

The density function given by (2) can be compactly represented by three pdf vectors $\tilde{P}_L$, $\tilde{\sigma}_L$, $\tilde{m}_L$ of probabilities, standard deviations, and means as shown below:

$$\begin{aligned}\tilde{P}_L &= \{P_{NL},\alpha_{L1},\alpha_{L2},\ldots,\alpha_{LK}\}\\ \tilde{\sigma}_L &= \{\sigma_{NL},\sigma_{pL1},\sigma_{pL2},\ldots,\sigma_{pLK}\}\\ \tilde{m}_L &= \{0,m_{pL1},m_{pL2},\ldots,m_{pLK}\}.\end{aligned} \tag{3}$$

As mentioned before, the mean vector $\tilde{m}_L$ is usually zero except in the case of wrong route blunder.

B. Navigation System Failure

Failures causing total loss of accurate navigation are modeled generally as a blunder and the vector description of the pdf is in the form of (3). If the navigation system L has failed by time T, then navigation reverts to air data system, and the errors grow as in the air data system from the start of failure.

The failure time T is discretized such that $T = N\Delta T$ (ΔT is typically 0.5 h). Given that a failure has occurred by time T, it is equally likely to have failed in any one of the intervals, $[0,\Delta T],[\Delta T,2\Delta T],\ldots,[(N-1)\Delta T,N\Delta T]$. Thus, the conditional failure probability vector is given by,

$$\tilde{P}_{bFL} = \left\{\frac{1}{N},\frac{1}{N},\ldots,\frac{1}{N}\right\} \tag{4}$$

where the index b stands for blunder and note that $\sum_{k=1}^{N} P_{bFLk} = 1$.

If a failure occurs at time $t = \lambda\Delta T$ ($\lambda = 1,2,\ldots,N$), then the error drifts off as an air data dead reckoning system after time $\lambda\Delta T$. The error growth, $\sigma_{AD}(t)$, for an airdata dead reckoning system for $t \geq \lambda\Delta T$ is given by,

$$\sigma_{AD}(t) = [(VLBAD)^2t^2 + (VLRWA)^2t]^{1/2} \tag{5}$$

where $VLBAD$ is the rms velocity and $VLRWA$ is the random walk error of air data. A standard deviation vector $\tilde{\sigma}_{bFL}$ can now be defined by

$$\begin{aligned}\tilde{\sigma}_{bFL} = \{&\sigma_{AD}(T),\sigma_{AD}(T-\Delta T),\sigma_{AD}(T-2\Delta T),\\ &\times\ldots,\sigma_{AD}(T-(N-1)\Delta T)\}.\end{aligned} \tag{6}$$

The mean vector $\tilde{m}_{bFL}$ is zero.

The probability of failure $\mathcal{P}_{bFL}$ can be determined by assuming that the failure characteristic of a navigation system follows the exponential law. By further assuming that the mean time to failure TF_L of the navigation system L is much greater than the duration of flight time, this probability can be approximated by

$$\mathcal{P}_{bFL} = \frac{T}{TF_L}. \tag{7}$$

The failure probability vector $\tilde{P}_{FL}$ can be obtained by multiplying $\tilde{P}_{bFL}$ by $\mathcal{P}_{bFL}$. Thus,

$$\tilde{P}_{FL} = \left\{\frac{1}{N},\frac{1}{N},\ldots,\frac{1}{N}\right\}\left(\frac{T}{TF_L}\right). \tag{8}$$

The standard deviation vector $\tilde{\sigma}_{FL}$ is given by

$$\begin{aligned}\tilde{\sigma}_{FL} &= \tilde{V}_F(\tilde{\sigma}_{bFL},\sigma_{NL})\\ &= \left\{\sqrt{\sigma_{bFL1}^2+\sigma_{NL}^2},\sqrt{\sigma_{bFL2}^2+\sigma_{NL}^2},\ldots,\sqrt{\sigma_{bFLN}^2+\sigma_{NL}^2}\right\}\end{aligned} \tag{9}$$

where $\tilde{V}_F(\cdot)$ has been defined as a vector functional of its arguments, σ_{bFLi}s are the components of the vector $\tilde{\sigma}_{bFL}$, and σ_{NL} may depend upon the time of failure $i\Delta T$. The mean vector $\tilde{m}_{FL}$ is zero.

Most transoceanic aircraft are equipped with more than one navigation system. In the case of aircraft having dual navigation systems, one of them is arbitrarily chosen as the primary navigator. If one fails, operation is automatically switched to the other. If one degrades, procedures are instituted to determine which one has degraded. This will give rise to the probability $PMSL$ of missing a degradation. As a consequence, the probability vector $\tilde{P}_L$ in (3) will change with no change in the standard deviation vector $\tilde{\sigma}_L$. This is discussed more fully in the next section. The case of total failure of both systems is now modeled.

Failure occurs in the case of dual systems only when both systems have failed and the failure rate is no longer uniform. Given that a failure has occurred by the time $T = N\Delta T$, the probability of the system having failed during the time interval $[(\lambda-1)\Delta T,\lambda\Delta T]$ is given by

$$P\{\text{failure in } [(\lambda-1)\Delta T,\lambda\Delta T\} = (2\lambda-1)/N^2.$$

Hence, the probability vector $\tilde{P}_{bFL}$ for the dual system is defined by

$$\tilde{P}_{bFL} = \left[\frac{1}{N^2},\frac{3}{N^2},\frac{5}{N^2},\ldots,\frac{2N-1}{N^2}\right]. \tag{10}$$

The probability of failure $\mathcal{P}_{bFL}$ before the observation time T is given by

$$\mathcal{P}_{bFL} = \frac{T^2}{TF_L^2}. \tag{11}$$

The standard deviation vector for the dual system remains the same as that of a single system, namely, the functional $\tilde{V}_F(\tilde{\sigma}_{bFL},\sigma_{NL})$ and the mean vector is zero. The failure probability vector $\tilde{P}_{FL}$ as given by (8) becomes

$$\tilde{P}_{FL} = \left[\frac{1}{N^2},\frac{3}{N^2},\frac{5}{N^2},\ldots,\frac{2N-1}{N^2}\right]\left(\frac{T^2}{T_{FL}^2}\right). \tag{12}$$

The pdf vectors given by (3) have to be modified when navigation system failures are also present. From assumption 1, the failure pdf vectors $\tilde{P}_{FL}$, $\tilde{\sigma}_{FL}$, $\tilde{m}_{FL}$ have to be added to the pdf vectors $\tilde{P}_L$, $\tilde{\sigma}_L$, $\tilde{m}_L$ of (3) resulting in pdf vectors $\bar{P}_{pL}$, $\bar{\sigma}_{pL}$, $\bar{m}_{pL}$ as shown below:

$$\begin{aligned}\bar{P}_{pL} &= \{P_{NL},\alpha_{L1},\alpha_{L2},\ldots,\alpha_{LK},\bar{P}_{FL}\}\\ \bar{\sigma}_{pL} &= \{\sigma_{NL},\sigma_{pL1},\sigma_{pL2},\ldots,\sigma_{pLK},\bar{\sigma}_{FL}\}\\ \bar{m}_{pL} &= \{0,m_{pL1},m_{pL2},\ldots,m_{pLK},\bar{m}_{FL}\}.\end{aligned} \tag{13}$$

The probability of normal navigation P_{NL} can be calculated from

$$P_{NL} = 1 - \sum_{j=1}^{K}\alpha_{Lj} - \sum_{j-1}^{N} P_{FLj}$$

where P_{FLj} is the jth component of the vector $\bar{P}_{FL}$. However, using the fact that $\sum_{j=1}^{N} P_{FLj} = \mathcal{P}_{bFL}$, the above equation can be simplified to

$$P_{NL} = 1 - \sum_{j=1}^{K}\alpha_{Lj} - \mathcal{P}_{bFL}. \tag{14}$$

C. Blunders

Assuming that there are no navigation errors, the conditional blunder pdf conditioned on a particular blunder J[1] can be represented by a weighted sum of Gaussian density function (assumption 2) as shown in (15).

$$f_b(x \mid \text{blunder } J) = \sum_{k=1}^{M}\beta_{Jk}\frac{1}{\sqrt{2\pi}\sigma_{bJk}}\exp\left[-\frac{(x-m_{bJk})^2}{2\sigma_{bJk}^2}\right] \tag{15}$$

where the summation over all β_{Jk} is 1, namely, $\sum_{k=1}^{M}\beta_{Jk} = 1$.

As in the previous cases ((3), (4), (6)), (15) can again be compactly represented by the three vectors as shown below:

$$\begin{aligned}\bar{P}_{bJ} &= \{\beta_{J1},\beta_{J2},\beta_{J3},\ldots,\beta_{JM}\}\\ \bar{\sigma}_{bJ} &= \{\sigma_{bJ1},\sigma_{bJ2},\sigma_{bJ3},\ldots,\sigma_{bJM}\}\\ \bar{m}_{bJ} &= \{m_{bJ1},m_{bJ2},m_{bJ3},\ldots,m_{bJM}\}.\end{aligned} \tag{16}$$

The index M represents the number of terms in building the blunder model with β_{Jk} being the weighting coefficient associated with each term with the sum over all k being equal to 1. The standard deviations and the means of individual terms are σ_{bJk} and m_{bJk}. These parameters may be implicit functions of time as in (6), and their functional form is developed in the next section. The probability of Jth blunder is represented by $\mathcal{P}_{bJ}$. The probability of no blunder in navigation system L is

Probability (no blunder in navigation system L)

$$= 1 - \sum_{i=1}^{J}\mathcal{P}_{bi}.$$

[1]The index J stands for either the Jth blunder or the total number of blunders in the navigation system L.

The overall blunder probability vector $\bar{P}_b$ can now be given by

$$\bar{P}_b = \{\bar{P}_{b1}\mathcal{P}_{b1},\bar{P}_{b2}\mathcal{P}_{b2},\ldots,\bar{P}_{bJ}\mathcal{P}_{bJ}\} \tag{17}$$

and the corresponding standard deviation and mean vectors are represented by

$$\begin{aligned}\bar{\sigma}_b &= \{\bar{\sigma}_{b1},\bar{\sigma}_{b2},\ldots,\bar{\sigma}_{bJ}\}\\ \bar{m}_b &= \{\bar{m}_{b1},\bar{m}_{b2},\ldots,\bar{m}_{bJ}\}.\end{aligned}$$

D. Total pdf for Navigation Errors and Blunders

The total pdf model when both navigation system errors including navigation system failure and blunders are present can now be derived. By assumption 3 the combined probability in the presence of J blunders in navigation system L is the product of the individual probability vectors. Thus, the total probability vector $\bar{P}_{TL}$ for the navigation system L is obtained by "multiplying" in the Kronecker sense the blunder probabilities given by (17) and the navigation probability vector $\bar{P}_{pL}$, as shown below:

$$\bar{P}_{TL} = \left\{\left[1-\sum_{i=1}^{J}\mathcal{P}_{bi}\right],\bar{P}_{b1}\mathcal{P}_{b1},\bar{P}_{b2}\mathcal{P}_{b2},\ldots,\bar{P}_{bJ}\mathcal{P}_{bJ}\right\}\{\bar{P}_{pL}\} \tag{18}$$

where the first term on the right-hand side of (18) represents the probability corresponding to no blunder. Note that the summation over all the components of the blunder vector $\bar{P}_{bi}$ is equal to 1 by definition. Note that the vector $\bar{P}_{pL}$ consists of the normal navigation probability P_{NL} and the degradation probabilities. Using assumption 3, the products of the probabilities of degradations and blunders can be neglected in the second and subsequent terms in the right-hand side of (18) and there results:

$$\bar{P}_{TL} = \left\{\left[1-\sum_{i=1}^{J}\mathcal{P}_{bi}\right]\bar{P}_{pL},\bar{P}_{b1}\mathcal{P}_{b1}P_{NL},\bar{P}_{b2}\mathcal{P}_{b2}P_{NL},\right. \\ \left.\times\ldots,\bar{P}_{bJ}\mathcal{P}_{bJ}P_{NL}\right\}. \tag{19}$$

An M-vector functional $\bar{V}_J$, as defined in (9), of the blunder standard deviation vector $\bar{\sigma}_{bJ}$ and the normal navigation error standard deviation σ_{NL}, is formed as shown in (20).

$$\bar{V}_J(\bar{\sigma}_{bJ},\sigma_{NL}) = \left\{\sqrt{\sigma_{bJ1}^2+\sigma_{NL}^2},\sqrt{\sigma_{bJ2}^2+\sigma_{NL}^2},\ldots,\sqrt{\sigma_{bJM}^2+\sigma_{NL}^2}\right\} \tag{20}$$

where $\bar{\sigma}_{bJ}$ and σ_{bJk} are defined in (16) and σ_{NL} in (2).

For the given navigation system L, the total pdf vectors for all J blunders can be now given by the following three pdf vectors, $\bar{P}_{TL}$, $\bar{\sigma}_{TL}$, $\bar{m}_{TL}$:

$$\bar{P}_{TL} = \left\{ \left[1 - \sum_{i=1}^{J} \mathcal{P}_{bi} \right] \bar{P}_{pL}, \bar{P}_{b1}\mathcal{P}_{b1}P_{NL}, \bar{P}_{b2}\mathcal{P}_{b2}P_{NL}, \times \ldots, \bar{P}_{bJ}\mathcal{P}_{bJ}P_{NL} \right\} \tag{21}$$

$$\bar{\sigma}_{TL} = \{\bar{\sigma}_{PL}, \bar{V}_1(\bar{\sigma}_{b1}, \sigma_{NL}), \bar{V}_2(\bar{\sigma}_{b2}, \sigma_{NL}), \ldots, \bar{V}_J(\bar{\sigma}_{bJ}, \sigma_{NL})\}$$

$$\bar{m}_{TL} = \{0, \bar{m}_{b1}, \bar{m}_{b2}, \ldots, \bar{m}_{bJ}\}$$

where $\mathcal{P}_{bi}$, which corresponds to α_{Lj} in (2) has been defined previously. The other terms are as defined in earlier equations.

E. Navigation Mixes

The above model can now be extended easily to cases where there are L different kinds of navigation systems in the air space. A quantity γ_L is defined as the ratio of the navigation system type L in the navigation mix operating in the air space. Under this condition, the final form of the probability model incorporating the γs is defined by the following three overall vectors, $\bar{P}_T$, $\bar{\sigma}_T$ and $\bar{m}_T$:

$$\begin{aligned} \bar{P}_T &= \{\gamma_1 \bar{P}_{T1}, \gamma_2 \bar{P}_{T2}, \ldots, \gamma_L \bar{P}_{TL}\} \\ \bar{\sigma}_T &= \{\bar{\sigma}_{T1}, \bar{\sigma}_{T2}, \ldots, \bar{\sigma}_{TL}\} \\ \bar{m}_T &= \{\bar{m}_{T1}, \bar{m}_{T2}, \ldots, \bar{m}_{TL}\} \end{aligned} \tag{22}$$

where all the vectors appearing in the right-hand side of (22) have been defined in (21).

III. DEVELOPMENT OF INDIVIDUAL ERROR MODELS

In the first part of this section, navigation system error models are developed for Inertial, OMEGA and Doppler navigation systems. The second part of this section develops pilot blunder models for waypoint insertion error, error due to pilot's inattention to autopilot, and wrong route error.

A. Navigation Error Models

Navigation error probability models for inertial, OMEGA, and Doppler navigation systems can be formulated within the framework of the general model developed in the previous section. The pdf vectors of dual inertial and OMEGA navigation systems have also been modeled.

Inertial Navigation System: The inertial system is a dead reckoning system whose errors as a function of time involves an Earth-rate loop of 24 h oscillations and a Schuler loop of 84 min oscillations. The pdf of the errors is a Gaussian whose parameters are the time-varying standard deviations caused by the two loop errors. The model used here simplifies the Earth loop to a random ramp and a random walk driven by the integral of the gyro drift rates. Schuler loop errors and accelerometer degradations have been neglected in this analysis. For an detailed discussion see [2, 9].

Thus at any specified time t, the pdf for the inertial system position errors conditioned under normal operation is a Gaussian density given by

$$f_{NI}(x) = \frac{1}{\sqrt{2\pi}\sigma_{NI}} \exp\left[\frac{-x^2}{2\sigma_{NI}^2}\right] \tag{23}$$

where σ_{NI} is the time-varying standard deviation of inertial system position errors given by

$$\sigma_{NI}(t) = [(GYRBN)^2 t^2 + (GYRRWN)^2 t]^{1/2} \tag{24}$$

where $GYRBN$ and $GYRRWN$ show the bias and random walk components in normal operation. The probability of normal inertial operation is P_{NI}.

Additional error growth can be experienced due to the degradation of gyro error growth characteristics either initially or over a period of time. Thus, T hours after a gyro degradation, the standard deviation for the additional growth terms of position error is

$$\sigma_{DGI}(T) = [(GYRBD)^2 T^2 + (GYRRWD)^2 T]^{1/2} \tag{25}$$

where $GYRBD$ and $GYRRWD$ show the bias and random walk components after degradation. Thus the total standard deviation of position error T hours *after* a degradation is represented by

$$\sigma_{IGD}(t,T) = [\sigma_{NI}^2(T) + \sigma_{DGI}^2(t-T)]^{1/2}, \qquad t \geq T. \tag{26}$$

The probability of degradation of the gyro at $T = 0$ will be assumed to be $PIDG$.

A second type of degradation is the incorrect initialization of the inertial system at take-off. The total standard deviation associated with this error is

$$\sigma_{II}(t) = [\sigma_{NI}^2(t) + (SINIT)^2]^{1/2} \tag{27}$$

where $SINIT$ is the standard deviation of the initial condition error. It is assumed that the probability of incorrect initialization is $PINIT$.

In order to make (26) more tractable, it is assumed that degradation occurs at $T = N \cdot DEL$ hours. At each DEL the probability of gyro degradation is assumed to be $DEL/TDIG$ with $TDIG$ being the mean time for gyro degradation.

The total pdf model for the inertial system position errors can now be written as follows:

$$f_I(x,t) = \frac{P_{NI}}{\sqrt{2\pi}\sigma_{NI}(t)}\exp\left[-\frac{x^2}{2\sigma_{NI}^2(t)}\right] + \frac{PINIT}{\sqrt{2\pi}\sigma_{II}(t)}\exp\left[-\frac{x^2}{2\sigma_{II}^2(t)}\right] + \frac{PIDG}{\sqrt{2\pi}\sigma_{IGD}(0,t)}\exp\left[-\frac{x^2}{2\sigma_{IGD}^2(t)}\right] + \sum_{K=1}^{N-1}\left(\frac{DEL}{TDIG}\right)\frac{1}{\sqrt{2\pi}\sigma_{IGD}(K\cdot DEL,t)} \times \exp\left[-\frac{x^2}{2\sigma_{IGD}^2(K\cdot DEL,t)}\right]. \quad (28)$$

Adding the failure pdf vectors $\bar{P}_{FI}$ and $\bar{\sigma}_{FI}$, the pdf vectors $\bar{P}_{pI}$ and $\bar{\sigma}_{pI}$ corresponding to (13) can be written as

$$\bar{P}_{pI} = \left\{P_{NI}, PINIT, PIDG, \frac{DEL}{TDIG}, \frac{DEL}{TDIG}, \ldots, \frac{DEL}{TDIG}, \bar{P}_{FI}\right\}$$
$$\bar{\sigma}_{PI} = \{\sigma_{NI}(t), \sigma_{II}(t), \sigma_{IGD}(0,t), \sigma_{IGD}(DEL,t), \times \ldots; \sigma_{IGD}[(N-1)DEL,t], \bar{\sigma}_{FI}\}. \quad (29)$$

The probability of normal operation of inertial system, P_{NI}, is calculated from (14):

$$P_{NI} = 1 - PINIT - PIDG - \frac{(N-1)DEL}{TDIG} - \mathcal{P}_{bFI}. \quad (30)$$

The mean vector $\bar{m}_{pI}$ is zero.

OMEGA Navigation Systems: Conditioned upon normal operation the OMEGA error model is a Gaussian pdf similar to (23) given by

$$f_{\rm NO}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\rm NO}}\exp\left[\frac{-x^2}{2\sigma_{\rm NO}^2}\right] \quad (31)$$

with $\sigma_{\rm NO}$ as the standard deviation of position errors under normal operation (NO) and probability of normal operation being $P_{\rm NO}$. Additional errors can occur due to lane slip (LS) and sudden ionospheric disturbance (SID). Normally, an LS would represent a deterministic bias error with a component in the lateral direction. However as a simplification, the pdf conditioned on LS is modeled as a random Gaussian bias such that the standard deviation after an LS is $\sqrt{\sigma_{\rm NO}^2 + \sigma_{\rm LS}^2}$ where $\sigma_{\rm LS}$ is the standard deviation of lane slip errors (typically 14 nm). It is also assumed, somewhat pessimistically, that once an LS occurs it persists for the remainder of the flight. The probability of an LS during flight is governed by an exponential distribution. However, under the assumption that the mean time for LS, TDO is much greater than the duration of the flight, this probability can be given by,

$$\Pr(\rm LS) \approx \frac{t}{TDO}. \quad (32)$$

In addition, there is a certain probability of LS at take-off given by $PRLSTO$. Thus, the total probability of LS is

$$P_{\rm LS}(t) = \frac{t}{TDO} + PRLSTO. \quad (33)$$

A second error source is the SID. This is governed by the Gaussian pdf with standard deviation $\sigma_{\rm SID}$. The probability of SID is taken as PSID. Thus the probability of normal operation $P_{\rm NO}$ can be given by

$$P_{\rm NO} = 1 - \left(\frac{t}{TDO} + PRLSTO\right) - PSID. \quad (34)$$

The total probability density function for the errors in an OMEGA system including navigation system failures at any given t can be represented by a probability vector $\bar{P}_{pO}$ and standard deviation vector $\bar{\sigma}_{pO}$, and (13) takes the form:

$$\bar{P}_{pO} = \{P_{\rm NO}, P_{\rm LS}(t), P_{\rm SID}, \bar{P}_{FO}\}$$
$$\bar{\sigma}_{PO} = \left\{\sigma_{\rm NO}, (\sigma_{\rm NO}^2 + \sigma_{\rm LS}^2)^{1/2}, \sigma_{\rm SID}, \bar{\sigma}_{FO}\right\}. \quad (35)$$

The mean vector $\bar{m}_{pO}$ is zero.

Doppler Navigators: Doppler navigators are not MNPS approved. However, a few slip into MNPS airspace and hence they have to be modeled. The Doppler navigator is modeled in a manner similar to the inertial navigation system (INS) navigator. The linear growth errors are caused by Doppler calibration errors and heading error. The random walk error terms are related to errors in processing the Doppler returns.

The standard deviation of Doppler errors under normal operation are:

$$\sigma_{ND}(t) = [(VLBIAN)^2 t^2 + (ERGTHN)^2 t]^{1/2}$$

where $VLBIAN$ is the ramp error growth term and $ERGTHN$ is the random walk growth term for normal operation. Because so few of them are present in the airspace, the degradation and failure effects have been ignored. For detailed analysis see [2, 9]. The pdf vectors corresponding to (13) are

$$\bar{P}_{pD} = [1]$$
$$\bar{\sigma}_{pD} = [\sigma_{ND}(t)]. \quad (36)$$

Other navigation error sources like dual GPS, INS/Omega and air data have also been modeled. For a detailed discussion, see [2, 9].

Modeling Dual Systems: As mentioned earlier, only the probability vector $\bar{P}_{pI}$ is modified for dual systems. The probability of missing a degradation for an inertial system is defined by $PMSI$. Factoring this probability in (29), the probability vector $\bar{P}_{pII}$ for a dual INS can be written as

$$\bar{P}_{pII} = \left\{P_{NII}, PMSI\left[PINIT, PIDG, \frac{DEL}{TDIG}, \frac{DEL}{TDIG}, \times \ldots, \frac{DEL}{TDIG}\right], \bar{P}_{FII}\right\} \quad (37)$$

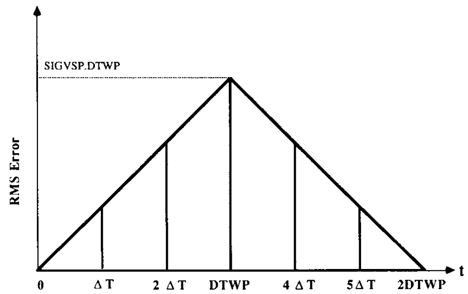

Fig. 1. Waypoint insertion error curve.

where

$$P_{NII} = 1 - PMSI\left[PINIT + PIDG + \frac{(N-1)DEL}{TDIG}\right] - \mathcal{P}_{bFII}$$

and the vector $\bar{P}_{FII}$ represents the conditional failure probability consistent with the dual system as defined in (12) and $\mathcal{P}_{bfII}$ is the failure probability similar to that defined in (11). The standard deviation vector $\bar{\sigma}_{PII}$ is as defined in (29) and the mean vector $\bar{m}_{PII}$ is zero.

In a similar manner the probability of missing a degradation in a dual OMEGA system is defined by $PMSO$ and the probability vector $\bar{P}_{pOO}$ is written as

$$\bar{P}_{pOO} = \{P_{NOO}, PMSO[P_{\mathrm{LS}}(t), P_{\mathrm{SID}}], \bar{P}_{FOO}\} \tag{38}$$

where

$$P_{NOO} = 1 - PMSO[P_{\mathrm{LS}}(t) + P_{\mathrm{SID}}] - \mathcal{P}_{bFOO}.$$

The standard deviation vector $\bar{\sigma}_{POO}$ is as defined in (35) and the mean vector is $\bar{m}_{POO}$ is zero.

B. Pilot Blunder Models

The navigation errors described earlier get augmented by human errors. Three types of human errors are modeled: 1) waypoint insertion error, 2) pilot's inattention to autopilot, and 3) wrong route error.

Waypoint Insertion Error: This is the most frequent error where the pilot inserts the wrong waypoint into his navigation computer. It can be characterized as a high speed (say 100 kt) departure from the planned track.

Generally, there is a waypoint at every 10 deg longitude, and at the latitudes of the North Atlantic there is a waypoint for every 300 nm. At 600 kt velocity it takes about one-half hour to reach the maximum error. The waypoint insertion error is modeled as a symmetric triangle-shaped error curve as shown in Fig. 1, which simulates a single waypoint error where the pilot heads away from the near waypoint and, after reaching $DTWP$ heads back to the next correct waypoint.

The time to maximum rms error is defined by $DTWP$. This time is discretized every ΔT hours such that $N = DTWP/\Delta T$ is an integer. $SIGVWP$ is defined as the standard deviation (usually 100 kt) associated with the cross-track velocity to reach the maximum rms error of $SIGVWP \cdot DTWP$ in $DTWP$ hours. $K\Delta T$ hours after the wrong waypoint insertion, the rms error is $SIGVWP \cdot K\Delta T$, and there are no errors at the endpoints. Since each state is equally probable, the symmetry of the situation leads to the vector description corresponding to (16) as shown below:

$$\bar{P}_{bW} = \left\{\frac{2}{2N-1}, \frac{2}{2N-1}, \ldots, \frac{1}{2N-1}\right\}$$

$$\bar{\sigma}_{bW} = \{SIGVWP \cdot \Delta T, SIGVWP \cdot 2\Delta T, \ldots, SIGVWP \cdot N\Delta T\}. \tag{39}$$

The mean vector $\bar{m}_{bW}$ is zero. Note that $\sum_{k=1}^{N} P_{bWk} = 1$.

How the waypoint insertion error vectors defined by (39) fit into the general model of (21) is discussed for an inertial navigator. The standard deviation of normal operation of an INS is σ_{NI} (24), and the corresponding probability is P_{NI} (30). The probability of a waypoint insertion error is $\mathcal{P}_{bW}$. The total pdf vectors given by the general model of (21) can be rewritten for an inertial navigator with waypoint insertion error as follows:

$$\bar{P}_{TIW} = \{[1 - \mathcal{P}_{bW}]\bar{P}_{pI}, \mathcal{P}_{bW}\bar{P}_{bW}P_{NI}\}$$
$$\bar{\sigma}_{TIW} = \{\bar{\sigma}_{pI}, \bar{V}_W(\bar{\sigma}_{bW}, \sigma_{NI})\} \tag{40}$$

where the vector functional $\bar{V}_W(\bar{\sigma}_{bW}, \sigma_{NI})$ is defined by

$$\bar{V}_W(\bar{\sigma}_{bW}, \sigma_{NI}) = \left\{\sqrt{\sigma_{bW1}^2 + \sigma_{NI}^2}, \sqrt{\sigma_{bW2}^2 + \sigma_{NI}^2}, \times \ldots, \sqrt{\sigma_{bWN}^2 + \sigma_{NI}^2}\right\}. \tag{41}$$

The mean vector $\bar{m}_{TW}$ is zero. Equation (40) is of the form of (21) and (41) is of the form of (20).

Pilot's Inattention to Autopilot: Occasionally, the pilot will inadvertently leave the autopilot in the wrong mode. This has been modeled as a slow ramp of the aircraft from track (typically 20–30 kt). Ultimately, the pilot will realize his error and gets back on track. The time of recovery of the pilot from this error is assumed to be governed by an exponential distribution $(1 - e^{-t/TR})$, where the mean time to recovery is defined by TR (typically one-half hour).

This model is similar to the waypoint insertion error model. Time is discretized every ΔT hours and to limit the number of terms, N is chosen to be an integer such that $N\Delta T = 3TR$. For convenience a quantity GG is defined by $GG = \Delta T/TR$. The probability that an error will occur due to pilot's inattention to autopilot is defined as $\mathcal{P}_{bAP}$. Thus, the probability that recovery will occur exactly at time $K\Delta T$ conditioned on the error due to pilot's inattention to autopilot is given by $e^{-(K-1)GG}(1-e^{-GG})$. Therefore, the blunder probability and the standard deviation vectors associated with this type of error are,

$$\bar{P}_{bAP} = \{1, e^{-GG}, e^{-2GG}, \ldots, e^{-(N-1)GG}\}(1-e^{-GG})$$
$$\bar{\sigma}_{bAP} = \{SIGVA \cdot \Delta T, SIGVA \cdot 2\Delta T, \ldots, SIGVA \cdot N\Delta T\} \tag{42}$$

where $SIGVA$ is the standard deviation (20–30 kt) of the induced cross-track velocity error. Note again that $\sum_{k=1}^{N} P_{bAPk} = 1$.

In a manner analogous to (40), (41) total probability and standard deviation vectors, $\bar{P}_{TIAP}$ and $\bar{\sigma}_{TIAP}$ for an inertial navigator with autopilot errors only are given by

$$\bar{P}_{TIAP} = \{[1-\mathcal{P}_{bAP}]\bar{P}_{pI}, \mathcal{P}_{bAP}\bar{P}_{bAP}P_{NI}\}$$
$$\bar{\sigma}_{TIAP} = \{\bar{\sigma}_{pI}, \bar{V}_{AP}(\bar{\sigma}_{bAP}, \sigma_{NI})\}. \tag{43}$$

The mean vector is zero and the vector functional $\bar{V}_{AP}(\bar{\sigma}_{bAP}, \sigma_{NI})$ is defined as usual by

$$\bar{V}_{AP}(\bar{\sigma}_{bAP}, \sigma_{NI}) = \left\{\sqrt{\sigma_{bAP1}^2 + \sigma_{NI}^2}, \sqrt{\sigma_{bAP2}^2 + \sigma_{NI}^2}, \times \ldots, \sqrt{\sigma_{bAPN}^2 + \sigma_{NI}^2}\right\}. \tag{44}$$

Wrong Route Error: Occasionally the pilot may fly a wrong route. This can be caused either by pilot error or an air traffic control loop error when the pilot misunderstands the clearance information given to him. Wrong route error is modeled as the error pdf of the aircraft having its mean at ± separation distance, $XSEP$, with a 50 percent probability of being one or the other given that a wrong route has occurred. Until now all pdfs associated with the various errors were assumed to have zero means and the probability of wrong route is defined as $\mathcal{P}_{bWR}$. As in the previous cases, the total pdf vectors for the inertial navigator with only wrong route errors can be written as follows:

$$\bar{P}_{TIWR} = \{[1-\mathcal{P}_{bWR}]\bar{P}_{pI}, \mathcal{P}_{bWR}\bar{P}_{bWR}P_{NI}\}$$
$$\bar{\sigma}_{TIWR} = \{\bar{\sigma}_{pI}, \sigma_{NI}, \sigma_{NI}\} \tag{45}$$
$$\bar{m}_{TIWR} = \{\bar{0}, +XSEP, -XSEP\}.$$

Total pdf Model Including All Blunders: In the previous three sections total pdf vectors were derived for inertial navigation system errors including navigation system failure with only a single blunder. The total pdf vectors can now be derived for all the three blunders (waypoint, autopilot, and wrong route) corresponding to (21).

$$\bar{P}_{TI} = \{[1-\mathcal{P}_{bW}-\mathcal{P}_{bAP}-\mathcal{P}_{bWR}]\bar{P}_{pI}, \mathcal{P}_{bW}\bar{P}_{bW}P_{NI}, \mathcal{P}_{bAP}\bar{P}_{bAP}P_{NI}, \mathcal{P}_{bWR}\bar{P}_{bWR}P_{NI}\}$$
$$\bar{\sigma}_{TI} = \{\bar{\sigma}_{pI}, \bar{V}w(\bar{\sigma}_{bW}, \sigma_{NI}), \bar{V}_{AP}(\bar{\sigma}_{bAP}, \sigma_{NI}), \sigma_{NI}, \sigma_{NI}\}$$
$$\bar{m}_{TI} = \{\bar{0}, \bar{0}, \bar{0}, +XSEP, -XSEP\}. \tag{46}$$

It is to be particularly noted that (46) is the final form of the total pdf vectors for an inertial navigation system with navigation system failure and waypoint, autopilot, and wrong route errors.

Overall Total pdf Model: The transoceanic air space will consist of many aircraft with different navigation systems. The ratio γ_L of each particular type of navigation has already been defined in an earlier paragraph (see (22)). In order to study the various critical parameters, the whole transoceanic space is treated as a system. Overall pdf vectors for the navigational system errors consisting of ratios of various navigation systems present in the airspace have been derived in (22). Thus, for the case of a mix of single inertial, single OMEGA, dual inertial, dual OMEGA and Doppler systems in the air space, the overall total pdf vectors are

$$\bar{P}_T = \{\gamma_I\bar{P}_{TI}, \gamma_{II}\bar{P}_{TII}, \gamma_O\bar{P}_{TO}, \gamma_{OO}\bar{P}_{TOO}, \gamma_D\bar{P}_{TD}\}$$
$$\bar{\sigma}_T = \{\bar{\sigma}_{TI}, \bar{\sigma}_{TI}, \bar{\sigma}_{TO}, \bar{\sigma}_{TO}, \bar{\sigma}_{TD}\} \tag{47}$$
$$\bar{m}_T = \{\bar{m}_{TI}, \bar{m}_{TI}, \bar{m}_{TO}, \bar{m}_{TO}, \bar{m}_{TD}\}$$

where the subscripts II, O, OO, and D represent the total pdf vectors corresponding to (46) for dual inertial, OMEGA, dual OMEGA, and Doppler navigation systems, respectively. Thus (47) is the complete causal probability model for the airspace carrying mixes of different navigation systems.

Thus the overall total pdf associated with the model including system failure and the various blunder probabilities can be written from (47) as follows:

$$f_T(x) = \frac{1}{\sqrt{2\pi}}\sum_{i=1}^{K}\frac{P_{Ti}}{\sigma_{Ti}}\exp\left[-\frac{(x-m_{Ti})^2}{2\sigma_{Ti}^2}\right] \tag{48}$$

where K is the total number of terms in the vectors.

Other Probability Functions: Two more probability functions based on (48) are now defined. The first is defined as the encroachment probability

$$PE(x) = \Pr(|X| > x).$$

In other words, this is the probability that the random variable X will be outside the corridor of width x. In terms of (48), this becomes

$$PE(x) = 2\sum_{i=1}^{K} P_{Ti}\text{erfc}\left[\frac{x-m_{Ti}}{\sigma_{Ti}}\right] \tag{49}$$

where erfc is the complementary error function. $PE(x)$ is used extensively in developing model parameters for the analysis described in later sections.

The second is the normalized overlap probability density. This term is directly proportional to the collision risk according to the Reich model [3–5]. In intuitive terms, it is the probability that two aircraft on adjacent tracks separated by $XSEP$ will overlap in the cross-track direction within one nm. Mathematically it can be described as

$$fo(XSEP) = \int_{-\infty}^{\infty} f(r + XSEP/2) f(r - XSEP/2)\, dr. \tag{50}$$

Assume that two aircraft on adjacent tracks have similar pdfs as described by (48) (except that one is displaced by $XSEP$ from the other). Then the normalized overlap probability[2] can be written, in terms of (48) as

$$fo(XSEP) = \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{P_{Ti} P_{Tj}}{\sqrt{\sigma_{Ti}^2 + \sigma_{Tj}^2}} \times \exp\left[-\frac{\{XSEP - (m_{Ti} - m_{Tj})\}^2}{2(\sigma_{TI}^2 + \sigma_{Tj}^2)} \right]. \tag{51}$$

In later sections this term is computed from the pdf for aircraft based on no ADS at $XSEP = 60$ nm. This value then defines a level of safety. Then the normalized overlap probability is recomputed after the impact of ADS on the system modeled. The distance $XSEP$, where the new overlap probability is equal to that computed for no ADS becomes the suggested reduced separations standard.

IV. PDF AFTER APPLICATION OF ADS

This section presents modifications of the overall total pdf model (eq. (3.27)) to reflect the presence of Automatic Dependent Surveillance (ADS). Two attributes of ADS are considered.

1. Sampling of aircraft position, and control of the aircraft when it strays from the prescribed track by a distance more than a specified threshold level.

This section presents modifications of the overall total pdf model (eqn. (48)) to reflect the presence of ADS. Two attributes of ADS are considered: 1) sampling of aircraft position, and control of the aircraft when it strays from the prescribed track by a distance more than a specified threshold level, and 2) failure or a degradation is indicated from the transmission of an FOM.

The first attribute drastically reduces the impact of pilot blunders. The second attribute reduces the impact of system failures and degradations. As a result of these two, the tails on the overall total pdf model of (48) will be significantly reduced, resulting in the possibility of lower separations standards.

[2]Note that m_{TI} or m_{Tj} is equal to 0 or $\pm XSEP$ in (51).

A. pdf After Sampling of Pilot Blunders

The pdf of the pilot blunder errors have to be determined after their sampling. The sampling of aircraft position identifies the wrong route error $\mathcal{P}_{bWR}$ and provides information for minimizing the impact of autopilot and waypoint insertion errors $\mathcal{P}_{bAP}$ and $\mathcal{P}_{bW}$. The sampling is so chosen that it is fast enough to minimize waypoint insertion errors and this also bounds the less severe autopilot error. The aircraft position error pdf is then determined after the sampling.

In the analysis the following assumptions are made. The sampling rate is 10/h. If the aircraft is detected as having strayed beyond 5 nm threshold value, it is commanded to return to track after a delay time of 1.2 min. In addition, there is 1 nm noise in measurement. The waypoint insertion error begins at a random phase with respect to the samples.

Based on the above assumptions a pdf function is numerically derived and a weighted least squares fit using Gaussian functions is performed [6]. The resultant pdf is described in terms of probability and standard deviation vectors $\bar{P}_{sbW}$ and $\bar{\sigma}_{sbW}$ which replace the vectors $\bar{P}_{bW}$ and $\bar{\sigma}_{bW}$ in (39). The numerical values of these vectors are

$$\bar{P}_{sbW} = \{0.0023, 0.5647, 0.281, 0.152, 0.68 \times 10^{-5}\}$$

$$\bar{\sigma}_{sbW} = \{1.52, 4.86, 8.87, 10.66, 18.65\}.$$

However, it should be pointed out that at the separation distances considered (30–60 nm), the effect on the overall pdf or overlap probability is essentially the same as setting $\mathcal{P}_{bW}$ to zero. Detailed analyses of the minimum sampling rate required is discussed in [6, 9].

B. Impact of Transmitting an FOM

It is currently anticipated that the ADS will contain an FOM message. The message will have a bit indicating redundancy, and a sequence of bits indicating "real time" quality of the navigation system. Two useful functions that can be served by the FOM transmission are: 1) identify failures (0 redundancy, 0 quality), and 2) provide an indication of system quality.

It is assumed in the modeling performed here that if either a failure or a degradation is indicated via the FOM, the aircraft is directed to a location of the airspace where it will be no threat to other aircraft. Modeling the impact of the FOM on the effects of failures is accomplished by setting the reliabilities of the navigators to a very high MTBF. Thus, no threat is attributed to failures.

The impact of degradations depends on the on-board algorithms used for degradation detection. The modeling used here is based on a simple test, involving the difference $|D|$ between the cross-track positions $X1$ and $X2$ indicated by the two navigators. If $|D|$ is greater than some threshold value KT, a degraded system is assumed to exist (and the FOM transmitted is changed accordingly). If $KT = 3.62\sigma$, where σ is the standard deviation of the normal system, the probability of false alarm has been calculated to be 0.01. The effects of the threshold detection scheme is next reflected in the pdf model. It should be noted that most commercial carriers actually carry three redundant systems, and a much more sophisticated error detection scheme is implemented. Thus, the results computed from the modeling described are quite conservative.

The encroachment probability, which has been introduced earlier (49), is now defined as the probability that the airccraft will deviate from its path by a specified amount without being detected by this threshold detection scheme. It is shown in [7] that the probability of encroachment with the dual system and the above parameters, is always less than that of a single system with a standard system encroachment probability (and pdf) and would be represented by a system with 1.8 times the normal error. This would be an upper (conservative) bound on the actual pdf.

The above-mentioned threshold detection scheme may not be effective in the case of OMEGA systems since the errors are likely to be common to both systems. However, it is assumed that one of the receiver status indicators (a light indicating ambiguity or "in the DR mode") would serve the same purpose for degradation detection, resulting in the same modeling procedures.

V. ESTIMATION OF KEY PARAMETERS

In order to use the model for evaluation, the following types of parameters are required:

1. mix of navigators: single INS, dual INS, single OMEGA, dual OMEGA, INS/OMEGA, and Doppler,
2. failure and degradation probability parameters,
3. normal and degraded error characteristics of each of the system types,
4. waypoint insertion and wrong route probabilities and characteristics,
5. autopilot inattention probabilities, and characteristics.

Data for the parameter estimation was obtained from three sources. First, the FAA provided data on the mix of navigation systems used on MNPS flights and the approximate number of flights per year. Secondly, manufacturer data was used when required. The third source was blunder data from NAT/SPG for the three-year period of 1983–1985. This generally included off-track error, reason for blunder, type of navigation system carried, and name of the carrier. Based on the information recorded, and some reasonable assumptions [8], each blunder was classified into one of six categories: 1) waypoint insertion error, 2) wrong route flown, 3) degraded system operation (but not total failure), 4) aircraft not MNPS equipped, 5) pilot inattention to autopilot mode, and 6) failures.

TABLE I
Breakdown of Blunder by Navigation System Type

Blunder Type	Total	INS/ INS	INS/ OMG	OMG/ OMG	Unk
Wrong Route	10	1	0	5	4
Waypoint	53	26	2	5	20
Degradation	10	3	1	4	2
No MNPS	29	2	0	7	20
Autopilot	6	4	0	1	1
Failure	18	5	0	13	0
Total	126	41	3	35	47
Subtotal Flight Path	63	27	2	10	34
Subtotal Other	63	14	1	25	23

Note: Aircraft cross-track deviation > 30 nm.

A breakdown of the blunder data by cause and type of navigation system is given in Table I. Generally, the approach used in establishing the parameters was to determine those sets of parameters which, when applied to the pdf model, would provide an expected number of encroachments equal to the number observed in the blunder data.

A. Aircraft Mix

Now the procedures used to establish the mix of aircraft are described. According to the data provided by the FAA, 90 percent of the aircraft carried dual INS, 5 percent carried dual OMEGAs, and 5 percent carried INS/OMEGA. Approximately 100,000 aircraft per year made the crossing, and the total aircraft for the three-year period was 300,000. Failure and degradation blunder data was not consistent with this mix of all dual systems.

For instance, using the pdf model described above it was determined that if the small percentage of aircraft carrying dual OMEGAs were to have the observed number of deviations greater than 30 nm caused by failure, the MTBF of each system in the dual configuration would have to be 35 h, which is not reasonable. Thus, it was hypothesized that dual OMEGAs have common modes of failure and were therefore modeled as single OMEGAs. Using the pdf model for single OMEGA it was determined that the number of 30 nm deviations computed would be the same as observed if these systems had an MTBF of 512 h. This is low but not unreasonable. *Note that it should be clearly understood that the authors intend that no inference be made relative to these values beyond that in using them in the model.*

A similar procedure [8] was used to determine the percent of single INS in the system. It was hypothesized that 4.5 percent of the population must have been operating with a single INS, where the

TABLE II
Input Parameters Into Model

Name	Parameter Description	Value	Comments
	For INS: Dual 85.5 percent, single 4.5 percent		
SHULN	Normal Schuler loop	0.32 nm	Error standard deviation
GYRRWN	Gyro-induced random walk	$0.44\ \mathrm{nm}/\sqrt{h}$	
GYRBN	Gyro-induced biased	1.3 nm/h	Velocity error, normal
GYRBN	Gyro-induced bias	4 nm/h	Velocity error, degraded
PIDG	Probability of initial degradation of gyro	0.004	~ 4 h with $TDGI = 1142$
*PMSI**	Probability of missing a degradation	0.05	For no ADS and dual systems
	For Omega: 5%		
SIGNO	Normal standard deviation	1.4 nm	
SIGDO	Sigma after LS	15 nm	
SIGSID	Standard deviation after SID	10 nm	
PMSO	Probability of missing a degradation	0.1	For no ADS and dual systems
	For Air Data		
VLBIAA	Bias velocity error	9.54	Mainly from heading error
	Blunder Data		
P_{bW}	Probability of waypoint	0.843×10^{-3}	
SIGVWP	Cross-track velocity after waypoint insertion	100 kt	
P_{bWR}	Probability of wrong route	0.000033	No ADS
		0	ADS
P_{bAP}	Probability of pilot inattention	0.0014	No ADS
		0	ADS

**Note*: Low value because most aircraft carry three systems.

MTBF of an INS is about 1500 h. The failure data for the mix of 5 percent INS/OMEGA was consistent with the parameters established for INS and OMEGA.

The last "mix" parameter needing a value is the fraction of the aircraft having Doppler navigators or equivalent systems which have improperly entered the airspace. There were 20 such instances reporting no INS or OMEGA navigation systems and were categorized as not MNPS equipped. It was assumed that they were all Doppler navigators drifting off at approximately 9.5 kt (rms). It can be shown that such a system will encroach 30 nm in 4 h with a probability 0.447. Thus, the fraction of Doppler navigators in the airspace was assumed to be

$$\text{fraction of Doppler navigators} = \frac{(20/300{,}000)}{0.447} = 0.00015.$$

B. Estimation of Degradation Parameters

Degradation parameters for INS were estimated as follows. For the given mix of single and dual systems, it was determined from the pdf model that the number of expected 30 nm deviations would equal the observed number, if the mean time to degradation of a gyro was 1142 h.

A similar procedure was used for the OMEGA system. It was determined that a good fit would occur if the mean time to LS (standard deviation 15 nm) were 560 h, and the probability of an SID (10 nm standard deviation) were 0.05.

C. Estimate of Pilot Blunder Parameters

The waypoint insertion error has been modeled as a triangular-shaped characteristic (Fig. 1) departing from track for 0.5 h at 100 kt (rms) and then returning at the same speed in 0.5 h. The velocity term was assumed to be a Gaussian random variable with a standard deviation of 100 kt.

Using the pdf model, it was determined that the probability of an encroachment being observed at the end of the flight, given that waypoint insertion error had occurred during the flight was 0.210. Fifty-three waypoint insertions were observed. Thus the probability of a waypoint insertion error, P_{bW}, could be computed as follows:

$$P_{bW} = (53/300000)/0.21 = 0.8413 \times 10^{-3}.$$

The probability of wrong route was determined by dividing the number observed by the total number of flights, that is

$$P_{bWR} = 10/300000 = 0.33 \times 10^{-4}.$$

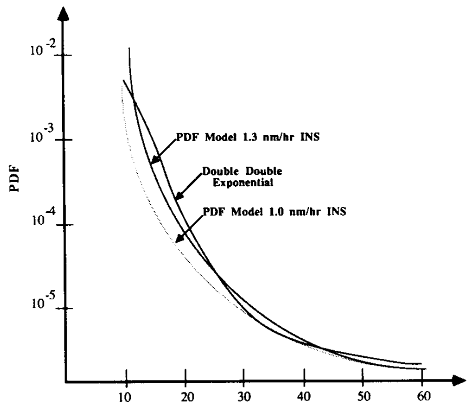

Fig. 2. Comparison of pdfs from double, double exponential and causal pdf models.

The probability of autopilot problems was computed by first assuming that the probability of such an error, P_{bAP} equals one. Then the model for the autopilot error was taken as a ramping off course at 30 kt rms, where there would be a mean time to recovery of 0.5 h. Under these assumptions, the probability of an encroachment is 0.014. The number observed is 6. Therefore, the P_{bAP} used in the model is

$$P_{bAP} = (6/300000)/0.014 = 0.0014.$$

Table II summarizes other important parameters which enter the pdf model.

VI. RESULTS AND SUGGESTIONS FOR REDUCING SEPARATION STANDARDS

The estimated parameters were inserted in the overall total pdf model to determine density functions and overlap probabilities. Fig. 2 shows the pdf function as derived from the pdf model, and its comparison to the reference double, double exponential (1). It is seen that there is a very close agreement between the two curves. This gives credibility to the overall pdf model developed in this paper.

Fig. 3 shows the computed encroachment probability for the overall pdf model versus distance from track. According to the pdf model there would be $0.431 \times 10^{-3} \times 300{,}000 = 129$ blunders (deviations greater than 30 nm) which are very close to the observed number of 126. However, it should be remembered that the parameters were specifically chosen so that the expected number would be very close to the observed 126.

Fig. 4 compares the normalized overlap probability of the pdf model and the double, double exponential versus the nominal separation distance. Again there is reasonably close agreement. This quantity is directly proportional to collision risk.

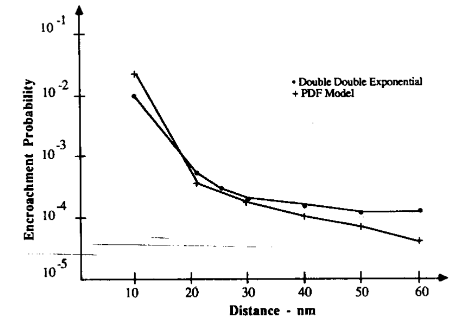

Fig. 3. Encroachment probability for 1.3 kt INS.

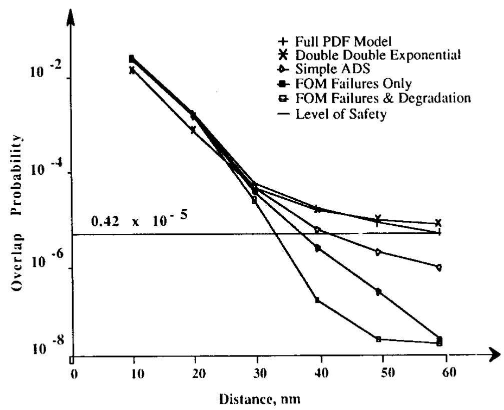

Fig. 4. Overlap probability for 1.3 nm.

In fact, the normalized overlap probability is used to define the level of safety. Assuming the validity of the overall pdf model, and that 60 nm separation is sufficient to minimize collision risk, then the level of safety is that value of overlap probability obtained from the model at 60 nm. This value is 0.42×10^{-5} as shown in Fig. 4.

Fig. 4 also compares the overlap probability versus separation distance for the total errors and for various errors eliminated. The separation distance where the normalized overlap probabilities cross the level of safety is the reduced lane separation possible. Although not shown here, the overlap probability if all Doppler navigators (non-MNPS aircraft) were eliminated from the population was computed. It was found that the overlap probability at 60 nm decreases by about 10 percent and the minimum separation distance reduces to 55 nm.

The overlap probability associated with a simple ADS performing position sampling shows that the minimum separation distance is about 41 nm. The model is modified as explained in Section IV. The pdf used assumes a sampling rate of 10/h. An alarm is sounded whenever the aircraft reports drifting off the prescribed path by more than 5 nm, and there is about 1.2 min delay in control of the aircraft.

The impact of transmitting an ADS FOM is considered next. A simple FOM reporting the presence of only a single INS at the start of the flight via a redundancy bit was assumed. Thus such aircraft would be eliminated from the population. Note that in this analysis it is still assumed that there are 5 percent single OMEGAs because it is believed that most of the OMEGA installations are dual, but their modes of failure are such that they can be modeled as single systems. In addition, the FOM would report system failure. This is modeled as described in Section IV. It is seen in Fig. 4 that the minimum separation distance drops to 36 nm.

When it is assumed that the FOM can also transmit information about degradations in the system as described in Section IV then the minimum separations standards can be further reduced to about 33 nm.

These results, based on the assumption that the normal INS navigation error drifted off at about 1.3 nm/h, were a closer fit to the reference double, double exponential, whereas, the stated INS errors are 1.0 nm/h rms (1σ). The same analysis as described above was also performed under the conditions that the normal INS error was 1 nm/h to determine minimum separations distances. Fig. 2 also compares the probability density curves for the total errors for 1.0 nm/h and 1.3 nm/h. Note that the results are nearly identical beyond 30 nm.

It is found that with a 1 nm/h INS it should be possible to achieve a separations standard below 30 nm, provided that the FOM transmitted can detect failures and degradations as outlined above.

Studies were also performed assuming that there was an ADS sampling rate of 20/h rather than 10/h. This, in effect, reduced the tails of the pdf arising after a waypoint insertion error and the reduction in separations standards from the 10/h case was less than 0.5 nm[3] which is not very significant.

VII. SUMMARY AND CONCLUSIONS

This paper has described the development of a causal probability model for cross-track excursions of transatlantic aircraft from its prescribed track for the conditions which exist today and for the situation where ADS will be applied. It factors in the mix and error characteristics of the navigators, their degradation characteristics and reliability. It also models specific types of pilot blunders. For the case of modeling when ADS is in place, it factors in the sampling and control characteristics and the use of degradation detection. It is a very detailed and flexible model allowing studies of what will result when some or all of the parameters which describe the overall system are changed.

A rational approach to defining the multitude of parameters based on available data from the North Atlantic region is also described. Much of this approach involves matching the number of aircraft observed encroaching 30 nm (for a specific reason) to the model derived encroachment probabilities (at 30 nm). It is shown that by using the model and the approach towards defining the parameters that the pdf very closely follows that derived from empirical data. This gives credibility to the model. The results for the case where ADS is in place have also been presented.

Numerical results are also presented for the mix of navigators currently in place in the North Atlantic route structure. The primary FOM in the analysis is the suggested lateral track separation distance which could provide the same level of safety as computed for the current system (mix of navigators and frequency of blunders) when various modifications are made to the system. The following are the general conclusions based on these numerical results.

1) Lateral track separation can be reduced from 60 nm to around 40 nm if a simple ADS is applied.
2) Sampling rates of around 10/hour should normally be sufficient to achieve the benefits of ADS (with respect to lateral separation) for the mix of navigation systems currently flying the North Atlantic.
3) Implementation of a time-varying FOM will allow further reduction of separation standards to slightly more than 30 nm. If the general preponderance on INS systems have a one sigma accuracy of 1 nm/hour, separations can be reduced to below 30 nm.

Although not discussed here, additional analysis using the model was performed for the case where GPS navigators were primarily used [2, 6]. The analysis indicated that separation distance of 15 nm or below could be used provided that the sampling rates were at least doubled.

Hopefully, the models and results here will provide additional impetus to the development of ADS and the subsequent reductions in separation standards.

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[3]It should be noted, however, that a higher sampling rate will be beneficial if an improved navigation is implemented and separations standards are allowed to approach 15 nm [6].

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